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A STATISTICAL PROCEDURE FOR DETERMINING BATTLE OUTCOME UTILIZING SEMI-MARKOV PROCESS

Peter C. C. Wang

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NAVAL POSTGRADUATE SCHOOL
Monterey, California

Rear Admiral Mason Freeman
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Jack R. Borsting
Provost

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This paper presents a stochastic approach to a Combat Model of tanks vs. tanks with emphasize of the following:

1. A scheme for development of trail validation criteria.
2. A useful procedure that can be expanded to a Semi-Markov model for combat
3. Approaches to data presentation.
4. Insight into data analysis and data reduction.

I. INTRODUCTION

As weapons systems grow in complexity, the cost in time and money of developing and testing advanced weapons is so great that military planners must insure that testing procedures for performance and tactics are formulated so as to obtain reliable data for each test. Thus the development of valid and reliable models for test design is a matter of great importance. Model manipulation is a far cheaper method for developing good system design than actual trial-and-error tests. This paper presents a stochastic approach to a combat model of tanks vs. tanks with emphasize on the following:

1. A scheme for development of trial validation criteria.
2. A useful procedure that can be expanded to a Semi-Markov model for combat (land, sea or air combat).
3. Approachs to data presentation.
4. Insight into data analysis and data reduction.

We let a point P, with coordinates (k,ℓ) represent the number of casulties of two opposing forces. Thus the coordinates are integers $0 \le k \le N$ and $0 \le \ell \le M$, where N is the size (number of units) of the friendly force and M is the size of the threat force. It is convenient to normalize this rectangle to the unit square, letting $x = k/n$, $y = \ell/m$ (see figure 1). The trial events are thus points on the unit square. Given a point (x,y) for any given weapon system there is some probability of transition to some other point (x_1,y_1), as the result of combat. The trial (battle) is over when either force becomes too small, that is, our point reaches some predetermined barrier on the top or on the right side of the square. It is our belief that if, in any real experiment (mock battle), one force quickly overwhelms the other, the experiment is a failure, in that it did not really test the system. Graphically the point (x,y) would move nearly vertically or nearly horizontally across the unit square. Thus we wish to study how to design the experiment so that the point (x,y) will, with some reasonable certainty, remain in a certain size cone about the 45° diagonal.

In describing our procedure, semi-Markov process and random walk terminology are used throughout. We are concerned with the development of the model and its ability to present data. In particular we will make certain simplifying assumptions concerning the transition probability matrix for illustrative purposes.

The technical portion of this paper seems new and provides a discrete state two-dimensional stochastic model of land, sea or air combat.

II THE MODEL

We assume that the duration of a trial is a random variable T and ΔT denotes the time interval used to update changes in systems' state (friendly and/or enemy kills). The realization of times where changes in system states are being updated are expressed in terms of:

$$0,\ \Delta T,\ 2(\Delta T),\ 3(\Delta T),\ 4(\Delta T),\ \ldots$$

ΔT will be selected based on the following factors:

1. ΔT cannot be selected too "small" due to limitations of the instrument used.

2. ΔT cannot be selected so "large" as to permit more than one system to be attrited from each force.

After ΔT is selected, the particle (representing the systems' state) may change its position or stay put as time progresses depending upon the results of some corresponding exchange of fire, but once it reaches a specified point on the lines $x = b$ and/or $y = b$ (where $50\% < b < 70\%$), the system or systems are considered to be saturated and the problem (engagement-trial) therefore terminated. This termination process compares with actual battle conditions where a specified force is considered noneffective when a percentage (50-70%) of casualties are sustained. (See Figure 1)

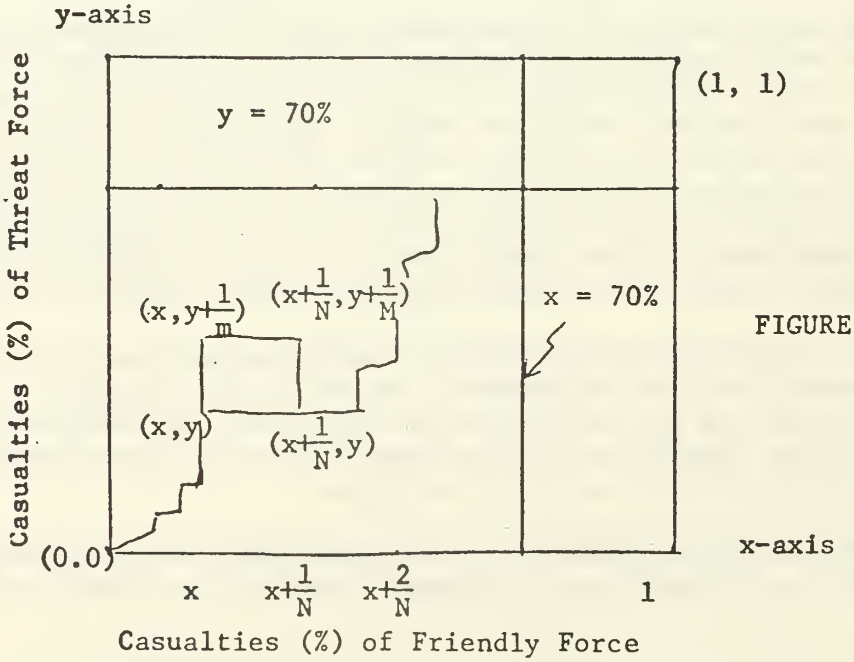


FIGURE 1.

At the commencement of the engagement (trial), time t, the system (friendly and aggressor) is in state (x,y). At the conclusion of a ΔT interval the position of the particle, say (x,y) moves to a

position denoted by S_y, S_x, and S_{xy} as outcomes of the ΔT time interval when the particle moves from (x,y) to $\left(x, y + \frac{1}{M}\right)$, $\left(x + \frac{1}{N}, y\right)$, $\left(x + \frac{1}{N}, y + \frac{1}{M}\right)$ respectively (see Figure 2). S_o can be used to further denote the status of the particle given no change in the state or increment of the particle. Based on the hypotheses used in this model, all the

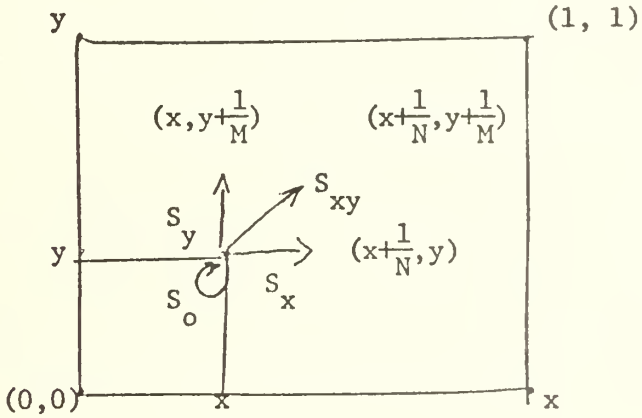


FIGURE 2.

possible positions of the particle are in the unit square and no outcomes are possible other than the above four, i.e., S_o, S_x, S_y, S_{xy}. However, instead of using the unit square as the sample space, it is easier to return to the rectangle Ω:

$$\Omega = \{(x, y);\ x = 0,1,2,\ldots,N;\quad y = 0,1,2,\ldots,M\}$$

Further, we define the sets:

$$A = \{(x,y):\ 0 \le x < N,\ y = M\}$$

$$B = \{(x,y):\ x = N,\ 0 \le y < M\}$$

$$C = \{(N,M)\}$$

$$I = \{(x,y):\ 0 \le x < N,\ 0 \le y < M\}$$

We shall call the totality of points in A or in B or in C the absorbing barriers, while all the points in I are called interior points.

For each point (x,y) in I, we assume we are given the probabilities:

$$P_r\{S_y\} = p,\ p > 0$$
$$P_r\{S_x\} = q,\ q > 0$$
$$P_r\{S_{xy}\} = r \text{ and } P_r\{S_o\} = s,\ r,\ s \geq 0$$

such that $p + q + r + s = 1$ and they are independent of T. For simplicity assume also they are independent of (x,y). In the actual experiment, these probabilities are functions of location (x,y). That is to say, the transition probabilities depend upon the remaining forces left in the battle zone. If one is interested in developing a stochastic model for a combat experiment, the probabilities p, q, r, and s will generally be functions of time, locations of the particle, and remaining units left in the battle zone.

For each point in the absorbing barriers, we assume that:

$$P_r\{S_o\} = 1,$$
$$P_r\{S_x\} = P_r\{S_y\} = P_r\{S_{xy}\} = 0.$$

Denote $P(x,y|t)$ the probability that the particle for the first time reaches the point (x,y) at time t and denote $\bar{P}(x,y|t)$ the probability that the particle is in state (x,y) at time t, and let $\tau(x,y)$ be the first passage-time through the point (x,y) namely, the number of (ΔT)'s to be performed until the particle for the first time reaches the point (x,y). Then, for any assigned values of N and M, $\tau(x,y)$ may be considered as a random variable, the realization of which equals one of the following values, i.e.,

$$t_o,\ t_o + 1,\ t_o + 2,\ \ldots,\ t_o + \nu, \ldots\ (0 \leq \nu < \infty)$$

where:

$$t_o = \max(x,y)\ .$$

Denote:

$$K(x,y,t) = \{k;\ \max(x + y - t,\ 0) \leq k < \min(x,y)\}$$

Denoting by $P_t(i,j,k,\ell)$ the probability function of the multinomial distribution $(p + q + r + s)^t$, the following well-known identity holds:

$$P_t(i,j,k,\ell) = \frac{t!}{i!j!k!\ell!} p^i q^j r^k s^\ell ; \quad (i+j+k+\ell = t)$$

and to any interior point (x, y), i.e., $(x,y) \varepsilon I$,

$$\hat{P}(x,y|t) = \sum_{k \varepsilon K} \frac{t!}{(s-k)!(y-k)!k!(t+k-x-y)!} q^{x-k} p^{y-k} r^k s^{t+k-x-y}$$

for $t = t_o, t_o + 1, t_o + 2, \ldots,$ where $t_o = \max(x,y)$.

For (x,y) and t such that $t < \max(x,y)$, it is clear that:

$$\hat{P}(x,y|t) = 0$$

For the first-passage probability $p(x,y|t)$ we have the following equations:

$$P(0,0|0) = 1 ,$$

$$P(x,y|t) = \hat{P}(x,y|t) - s\hat{P}(x,y|t-1) .$$

If $(x,y)\varepsilon I$, we should have:

$$P(x,y|t) = \sum_{k \varepsilon K(x,y,t)} \frac{(x+y-k)!}{(x-k)!(y-k)!\, k!} q^{x-k} p^{y-k} r^k \binom{t-1}{x+y-k-1} s^{t+k-x-y} ,$$

provided $t \geq t_o$.

Proofs of all formulas and optimal mixes of weapon systems used for both the friendly force and the threat force will be treated in a separate paper.

Denote by $\phi_t(u,v)$ the generating functions of:

$\hat{P}(x,y|t)$, namely

$$\phi_t(u,v) = \sum_{x=0}^{t} \sum_{y=0}^{t} \hat{P}(x,y|t)\, u^x v^y$$

Denote Φ as the generating function of ϕ_t, then:

$$(1) \qquad \Phi(u,v,\theta) = \sum_{t=o}^{\infty} \phi_t \, \theta^t$$

$$= \sum_{t=o}^{\infty} \theta_t \sum_{x=o}^{t} \sum_{y=o}^{t} \hat{P}(x,y|t) \, u^x \, v^g$$

$$= [1-(qu + pv + ruv + s)]^{-1}$$

The generating functions of $P(x,y|t)$ can be expressed as:

$$(2) \qquad \psi(u,v,\theta) = \sum_{x=o}^{\infty} \sum_{y=o}^{\infty} \left(\sum_{t=\max(x,y)}^{\infty} P(x,y|t)\theta^t \right) u^x \, v^y$$

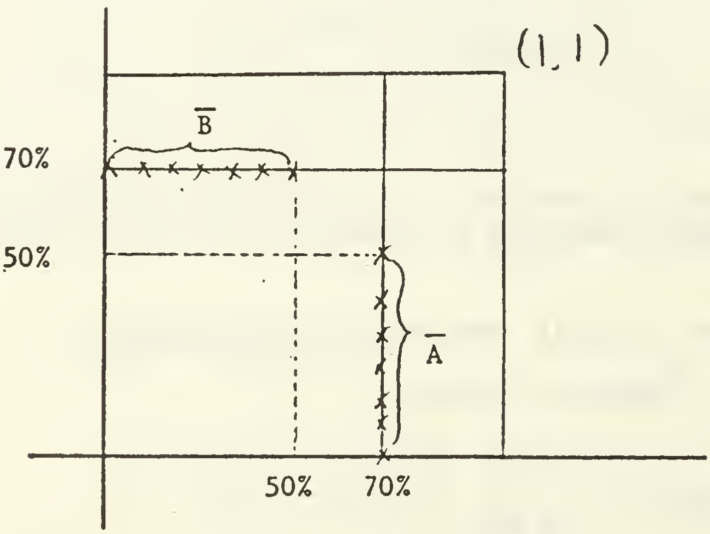
$$= \Phi(u,v,\theta) - s\theta \, \Phi(u,v,\theta)$$

$$= \left(1 - \frac{\theta}{1-s\theta} \, (qu + pr + ruv \right)^{-1}$$

It is to be noticed that if $(x,y) \notin I$, then the corresponding probabilities p and q in the above expressions do not give the required probabilities.

III. PROBABILITY FUNCTIONS FOR THE POINTS ON ABSORBING BARRIERS

Probabilities $P(x,y|t)$ and $\hat{P}(x,y|t)$ can be computed from formulas (1) and (2). In order to avoid early terminations of trials, these probabilities $P(\bar{A})$ and $P(\bar{B})$ should be computed and evaluated over sets $\bar{A}$ and $\bar{B}$ respectively, as illustrated by the following figure.



These probabilities over sets $\bar{A}$ and $\bar{B}$ should be very close to each other in order to warrant a good design (i.e., trials with adequate data points).

If $P(\overline{A})$ is considerably larger than $P(\overline{B})$, then an increase of friendly units (or decrease of the threat units) in the original mix is necessary to guarantee a successful trial. If $P(\overline{A})$ is considerably smaller than $P(\overline{B})$, similar policies can be adopted.

Denote $p(x,M|t)$ the absorption probability with respect to the point $(x,M)\varepsilon A$ and denote $p(N,y|t)$ the absorption probability with respect to the point $(N,y)\varepsilon B$ at the end of t-th (ΔT) trial. The expressions of these absorption probabilities are given below:

$$P(x,M|t) = \sum_{K(x,M,t)} \frac{M(t-1)!\, q^{x-k} p^{M-k} r^{k} s^{t-x-M+k}}{(x-k)!\,(M-k)!\,k!\,(t-x-M+k)!}, \tag{1}$$

provided $0 \le x < N$ and $M \ge 1$.

$$P(N,y|t) = \sum_{K(N,Y|t)} \frac{N(t-1)!\, q^{N-k} p^{y-k} r^{k} s^{t-N-y+k}}{(N-k)!\,(y-k)!\,k!\,(t-N-y+k)!} \tag{2}$$

provided $0 \le y < M$, $N \ge 1$.

$$P(N,M|t) = r \sum_{k(N-1,M-1,t-1)} \frac{(t-1)!\, q^{N-1-k} p^{M-1-k} r^{k} s^{t+1+k-N-M}}{(N-1-k)!\,(M-1-k)!\,k!\,(t-1+k-N+2-M)!} \tag{3}$$

provided $N \ge 1$, $M \ge 1$.

IV. ABSORPTION PROBABILITIES:

Let us denote $P_\alpha(N,M)$ the probability that the random walk will terminate at the point (α,M) and by $P_A(N,M)$ the probability that it will terminate at any point on the absorbing barrier A, then we have:

$$P_\alpha(N,M) = \sum_{t=\max(\alpha,M)} P(\alpha,M|t)$$

provided $0 \le \alpha < N$ and $M \ge 1$, and

$$P_A(N,M) = \sum_{\alpha=0}^{N-1} p_\alpha(N,M) \quad M \ge 1.$$

Denoting by $\tau(\alpha,M)$ the number of updatings to be performed until the particle will be absorbed at point $\tau(\alpha,M)$, the distribution of $\tau(\alpha,M)$ should be defined by the conditional probabilities as follows:

$$p(\tau(\alpha,M) = t) = \frac{p(\alpha,M|t)}{p_\alpha(N,M)}$$

for $t = \max(\alpha,M), \max(\alpha,M) + 1, \ldots$.

The expected number of updatings is:

$$E[\tau(\alpha,M)] = \sum_{t=\max(\alpha,M)}^{\infty} t \frac{p(\alpha,M|t)}{p_\alpha(N,M)} \tag{4}$$

The computation (4) is pretty combersome for the above quantity. Yet $E[\tau]$ is very important for us to know. If it turns out that $E[\tau]$, the expected number of updatings, is very small, we will have insufficient data during the trial. If $E[\tau]$ is large, then the selection of (ΔT) may be <u>too small</u> or the trial may last <u>too long</u>.

The following paragraphs will be devoted to a discussion how $E[\tau]$ can be used in computing the expected duration of the trial.

If $(\alpha,M)\varepsilon A$, the quantity:

$$\sum_{t=\max(\alpha,M)}^{\infty} t\, P(\alpha,M|t)$$

would be given as the coefficient of the term $u^\alpha v^M$ in the expansion of the following power series:

$$\frac{1 - (qu + s)}{1 - (qu + pv + ruv + s)}$$

Denote by $D_A(N,M)$ the expected duration of the random walk, assuming that it will terminate at any point on the absorbing barrier A. Then we have:

$$D_A(N,M) = \frac{1}{P_A(N,M)} \sum_{\alpha=0}^{N-1} E(\tau(\alpha,M))\, p_\alpha(N,M) \quad .$$

V. APPLICATION

Consider the rectangle illustrated in Figure 3. We wish to determine the mixes of forces, i.e., to determine N,M such that with high probability the particle will be absorbed in the absorption barrier in the set C* (see Figure 3) where:

$$C^* = \{(x,y): \text{ either } x = N, \frac{M}{2} \leq y \leq M \text{ or } y = M, \frac{N}{2} \leq x \leq N\} .$$

In other words, one can try to find various values of N,M such that the absorption probability has reached to some acceptable level (say 80%, i.e., P(C) = 80%). Once such an optimal mix has been found, experiments can be repeated with a change of test conditions, i.e., terrains or probability of kill tables, in order to obtain data which in turn will be used to detect any probability changes. As the combat between the friendly force and the threat force going on in the battle zone, one keeps a continuous record on the quantity defined as the exchange ratio at time t. Definition of Exchange Ratio at Time t (denote as ER(t)) is

$$ER(t) = \frac{\dfrac{\text{\# of threat tank loss up to and including time t}}{\text{total \# of threat tanks}}}{\dfrac{\text{\# of friendly anti-tank systems loss up to and include time t}}{\text{total \# of friendly anti-tank systems}}}$$

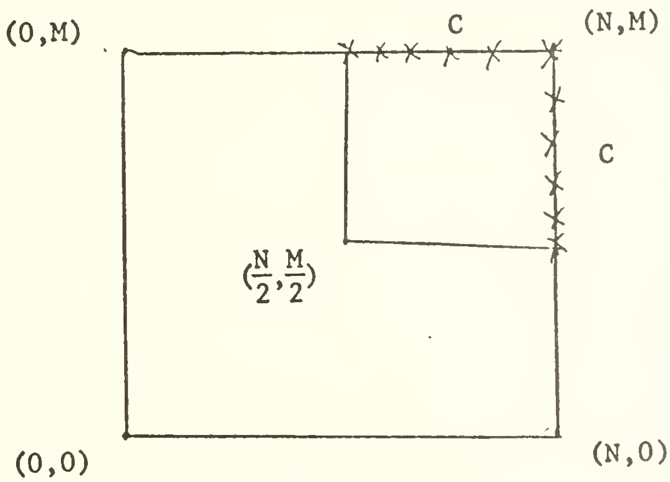
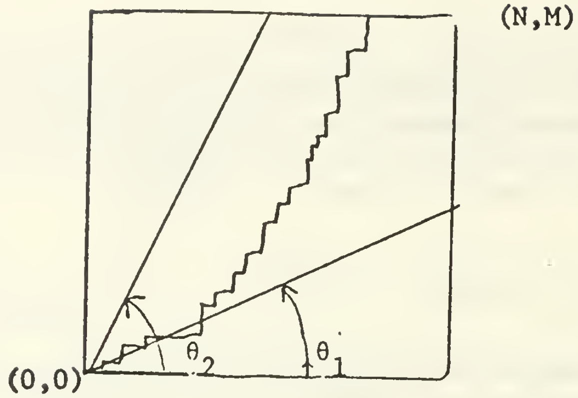


Figure 3

The above definition is used for "exchange ratio at time t" if data source and instruments are at high performance level. A typical graph of this exchange ratio ER(t) is given in Figure 4.

Figure 4.



In order to control the balance of force structures and mixes, it is ideal to have some bounds, say a and b such that the probability that ER(t) is bounded in between a and b for all $t > 0$ is very high, i.e.,

$$P_r\{a < ER(t) < b;\ 0 < t\} = .95$$

It is clear that $a = \tan \theta_1$ and $b = \tan \theta_2$ where θ_1 and θ_2 are shown in Figure 4.

Using a two-dimensional random walk model [2.4], we have illustrated above how to determine the best force structures such that the exchange ratio is close to one for most of the times during a trial. In other words, the probability that the sample path falls into the shaded region (see Figure 5) is maximized by an optimal choice of force structures.

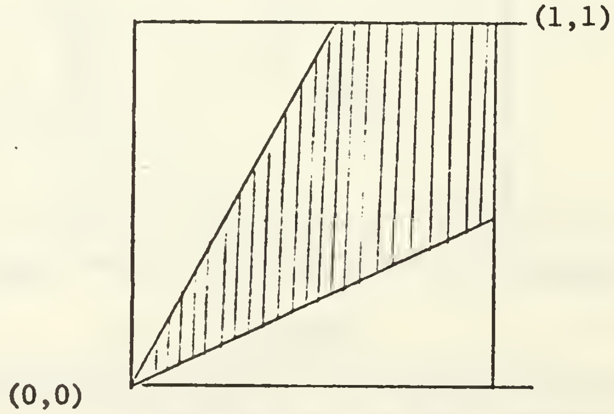


Figure 5

Due to the fact that instruments are most likely working at low performance levels, we provide an alternative definition for "exchange

ratio" using indirect measurements. This proposed definition of "exchange ratio" is the ratio of expected number of kills normalized by the initial force units.

The above definition of the exchange ratio is closer to the measure of effectiveness rather than the exchange of kill ratio but due to the instrument limitations this may be a good alternative measure of exchange ratio.

Another way of defining the same quantity is by looking at the remaining kill potential or fighting value of the forces. This remaining fighting force would be compared with their respective fighting potentials relative to their respective initial fighting potential. The degradation of their respective potentials over time will be plotted on a graph sheet for each trial.

The literature indicated that discrete state stochastic model formulations of combat have been difficult to solve even when the process is considered to be Poisson (Lanchester type) with stationary transistion mechanisms [1].

The technical portion of our discussion seems new and provides a discrete state two-dimensional stochastic model of combat which can be extended into a complete stochastic model for field experimentation such as the Tactical Effectiveness Testing of Anti tank Missiles [3].

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